

B. Tech.

SECOND SEMESTER EXAMINATION 2009-10

ENGINEERING MECHANICS

Time : 3 Hours

Total Marks : 100

Note : (i) This paper is in three sections. Section A carries 20 marks, Section B carries 30 marks and Section C carries 50 marks.

- (ii) Attempt all questions. Marks are indicated against each question part.
(iii) Assume missing data suitably, if any.

Section-A

Q. 1. You are required to answer all the parts :

Choose correct answer for the following parts.

(a) The necessary and sufficient condition for a system of coplanar forces to be in equilibrium.

(i) $\Sigma F_x = 0$

(ii) $\Sigma F_x = \Sigma F_y = 0$

(iii) $\Sigma M_0 = 0$

(iv) $\Sigma F_x = \Sigma F_y = \Sigma M_0 = 0$

(b) The bending equation is :

(i) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$

(ii) $\frac{M}{y} = \frac{\sigma}{I} = \frac{E}{R}$

(iii) $\frac{M}{y} = \frac{\sigma}{R} = \frac{E}{I}$

(iv) $\frac{M}{I} = \frac{\sigma}{R} = \frac{E}{y}$

(c) The principle of conservation of energy can't be applied in the following situation :

(i) body sliding down on a rough inclined plane.

(ii) simple pendulum

(iii) a particle executing SHM

(iv) a particle moving in a gravitational field

(d) In UDL loading (ω N/m), the maximum bending moment in case of simple supported beam is given as :

(i) ωL

(ii) $\frac{\omega L^2}{2}$

(iii) $\frac{\omega L^2}{4}$

(iv) $\frac{\omega L^2}{8}$

Ans. 1. (a) (i) $\Sigma F_x = 0$ (b) (ii) $\frac{M}{I} = \frac{\sigma}{y} = \frac{E}{R}$ (c) (d) (iv) $\frac{\omega L^2}{8}$

Fill in the blanks for the following parts :

You will be awarded full marks, if all the entries in a part are correct otherwise will be awarded zero.

(e) The algebraic sum of the moments of two forces with respect to any moment centre in their plane of action is equal to the moment of their with respect to the same centre.

(f) In a cantilever beam carrying a concentrated load at the free end, the bending moment will be zero at and maximum at

(g) The angular velocity (rad/sec) of a body rotating at N rpm is and the linear velocity of a body rotating at ω rad/sec along a circular path of radius r is

(h) In truss analysis, all forces acting on truss are applied at the only and also lie in the of truss.

Ans. (e) Coplaner forces with respect to any moment centre to the moment of their resultant force with respect to the same centre.

(f) The bending moment will zero at starting point and maximum at ending or where it is hinged.

(g) $\frac{2\pi N}{60}$ rad/sec, $r \frac{d\theta}{dt}$

(h) Joint only and also lie in the member of the truss.

Match the columns for the following parts :

You will be awarded full marks, if all the matches in a part are correct otherwise will be awarded zero.

(i) Match the following columns. Column II shows the moment of inertia about a centroidal axis :

| | Column I | | Column II |
|-------|------------|-----|---------------------|
| (i) | Triangle | (P) | $0.1 LR^4$ |
| (ii) | Circle | (Q) | $\frac{\pi R^4}{4}$ |
| (iii) | Semicircle | (R) | $\frac{bh^3}{12}$ |
| (iv) | Rectangle | (S) | $\frac{bh^3}{36}$ |

(j) Match the following columns :

| | Column I | | Column II |
|-------|----------------------|-----|---|
| (i) | Curvilinear motion | (P) | Neither pure rotation nor pure translation |
| (ii) | Rectilinear motion | (Q) | Pure rotary motion |
| (iii) | General plane motion | (R) | Motion of particles remains parallel and straight |
| (iv) | Instantaneous motion | (S) | Motion of particles remains parallel and in curve |

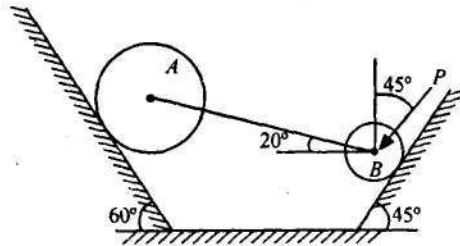
Ans. (i) (i) \rightarrow (S) (ii) \rightarrow (Q) (iii) \rightarrow (P) (iv) \rightarrow (R)
 (j) (i) \rightarrow (S) (ii) \rightarrow (Q) (iii) \rightarrow (R) (iv) \rightarrow (P)

Section-B

Q. 2. Answer any three parts of the following :

(a) Two cylinders A and B weighing 4 kN and 3 kN, respectively, rest on smooth inclined plane as shown in Figure 1. They are connected by a bar of negligible weight hinged to each cylinder at its

geometric centre by smooth pins. Find the force P to be applied to the smaller cylinder at 45° to the vertical to hold the system in the given position.



Ans.

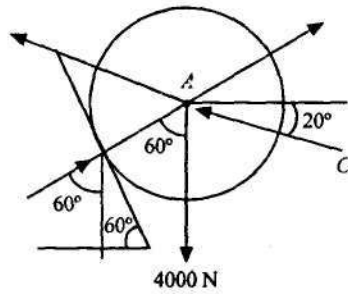


Fig. 1.

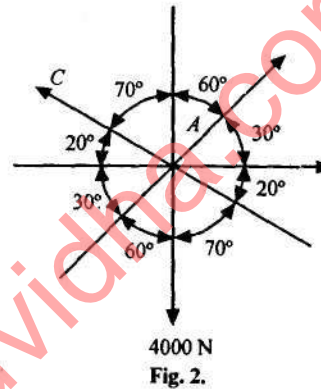


Fig. 2.

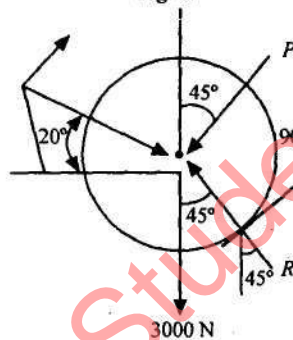


Fig. 3.

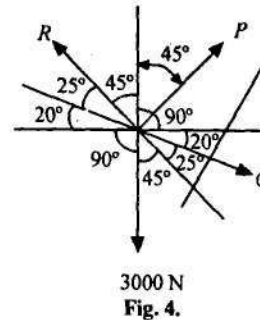


Fig. 4.

By the help of Lami's theorem from fig. 2.

$$\frac{C}{\sin(120^\circ)} = \frac{4000}{\sin(130^\circ)} = R$$

\therefore

$$C = 4522.06 \text{ N}$$

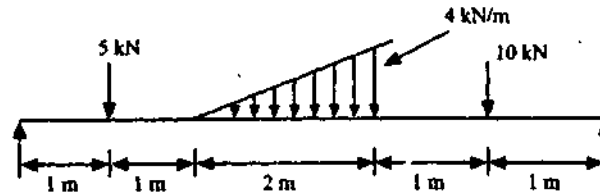
From fig. 3 summation of the forces parallel to the inclined plane (45° to horizontal) = 0

$$P + 3000 \cos 45^\circ - C \cos 45^\circ = 0$$

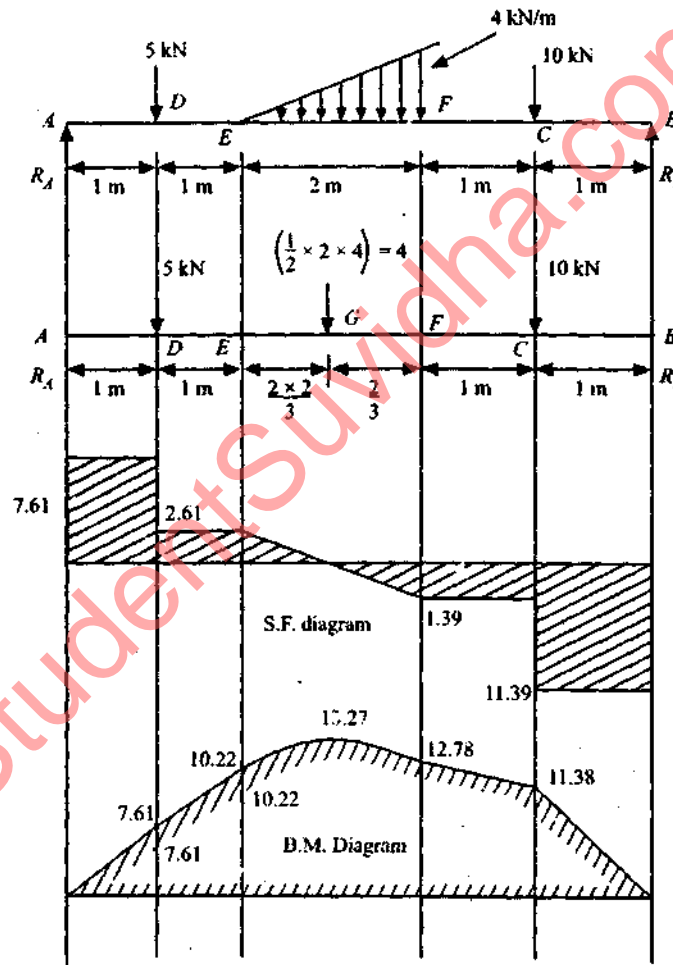
\therefore

$$\begin{aligned} P &= 4522.06 \cos 45^\circ - 3000 \cos 45^\circ \\ &= 1076.26 \text{ N} \end{aligned}$$

Q. 2(b). Calculate the values of shear force and bending moments for the simple supported beam shown in Figure. Also draw the shear force and bending moment diagrams.



Ans.



By the help of equilibrium condition

$$\begin{aligned}\sum F_v &= 0 \\ R_A - 5 - 4 - 10 + R_B &= 0 \\ R_A + R_B &= 19 \text{ kN}\end{aligned}$$

$$\sum M_A = 0$$

$$R_A \times 0 - 5 \times 1 - 4 \times \left(2 + \frac{4}{3}\right) - 10 \times (5) + R_B \times 6 = 0$$

\therefore

$$R_B = 11.39 \text{ kN}$$

$$R_A = 7.61 \text{ kN}$$

$$\text{S.F. between } AD = R_A = 7.61 \text{ kN}$$

$$\text{S.F. between } DE = 7.61 - 5 = 2.61 \text{ kN}$$

$$\text{S.F. between } GE = 2.61 \text{ kN}$$

$$\begin{aligned} \text{S.F. between } GF &= 2.61 - 4 \\ &= -1.39 \text{ kN} \end{aligned}$$

$$\text{S.F. between } FC = -1.39 \text{ kN}$$

$$\begin{aligned} \text{S.F. between } CB &= -1.39 - 10 \\ &= -11.39 \text{ kN} \end{aligned}$$

$$\text{S.F. at } B = -11.39 + 11.39 = 0$$

$$\text{B.m. at point } A = 0$$

$$\text{B.m. at point } B = 7.61 \times 1 = 7.61 \text{ kN.m}$$

$$\text{B.m. at point } E = 7.61 \times 2 - 5 \times 1 = 10.22 \text{ kN.m}$$

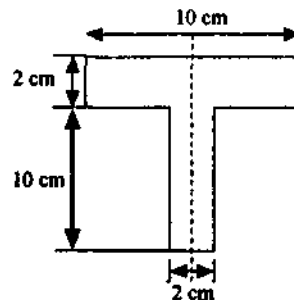
$$\begin{aligned} \text{B.m. at point } G &= 7.61 \times \left(2 + \frac{4}{3}\right) - 5 \times \left(1 + \frac{4}{3}\right) \\ &= 13.7 \text{ kN.m} \end{aligned}$$

$$\begin{aligned} \text{B.m. at point } F &= 7.61 \times (4) - 5 \times 3 - 4 \times \frac{2}{3} \\ &= 12.78 \text{ kN.m} \end{aligned}$$

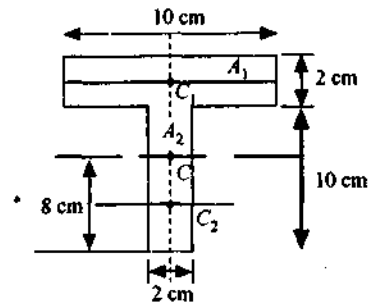
$$\begin{aligned} \text{B.m. at point } C &= 7.61 \times 5 - 5 \times 4 - 4 \times \left(\frac{2}{3} + 1\right) \\ &= 11.38 \text{ kN.M} \end{aligned}$$

$$\begin{aligned} \text{B.m. at point } B &= 7.61 \times 6 - 5 \times 5 - 4 \times \left(\frac{2}{3} + 2\right) - 10 \times 1 \\ &= 0 \text{ kN.M} \end{aligned}$$

Q. 2(c). Determine the moment of inertia of T section about the horizontal and vertical axes, passing through the C.G. of the section as shown Fig.



Ans.



| Figure | Area (mm ²) | x co-ordinate of the centroid (mm) | y co-ordinate of the centroid (mm) |
|-----------------|---------------------------------------|------------------------------------|------------------------------------|
| Rectangle A_1 | $A_1 = 2 \times 10 = 20 \text{ mm}^2$ | $x_1 = 0$ | $y_1 = 11 \text{ cm}$ |
| Rectangle A_2 | $A_2 = 2 \times 10 = 20 \text{ mm}^2$ | $x_2 = 0$ | $y_2 = 5 \text{ cm}$ |

$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

$$= \frac{20 \times 11 + 20 \times 5}{20 + 20} = 8 \text{ mm}$$

Area A_1 : M.I. of area A_1 about its centroidal x-axis (that is the axis through C_1)

$$I_{x_1} = \frac{1}{12} \times 10 \times (2)^3$$

$$= 667 \text{ mm}^4$$

M.I. of area A_1 about the centroidal x axis (that is the axis through C_2)

$$(I'_x)_1 = (I_{x_1}) + A_1 d^2$$

$$= 667 + 20 \times (11 - 8)^2$$

$$= 18667 \text{ mm}^4$$

Area A_2 : M.I. of the area A_2 about its centroidal axis (that is the axis through C_2)

$$(I_x)_2 = \frac{1}{12} \times 2 \times 10^3$$

$$= 16667 \text{ mm}^4$$

M.I. of the area A_2 about the centroidal x axis of the composite area

$$(I'_x)_2 = (I_x)_2 + A_2 d^2$$

$$= 16667 + 20 \times (8 - 5)^2$$

$$= 34667 \text{ mm}^4$$

$$\text{Composite area } I'_x = (I'_x)_1 + (I'_x)_2 = (18667 + 34667) \text{ mm}^4$$

$$= 51334 \text{ mm}^4$$

Q. 2(d). A solid shaft is subjected to a maximum torque of 15 MN-cm. Determine the diameter of the shaft, if the allowable shear stress and the twist are limited to 1 kN/cm² and 1°, respectively for 210 cm length of shaft. $G = 8 \text{ MN/cm}^2$.

Ans. $T = 15 \text{ MN-cm} = 15 \times 10^6 \times 10 \text{ N-mm} = 15 \times 10^7 \text{ N-mm} = 15 \times 10^3 \text{ kN-cm}$

Diameter based on shear stress :

$$\sigma_s = 1 \text{ kN/cm}^2$$

$$\frac{T}{J} = \frac{\sigma}{R}$$

$$\therefore \frac{15 \times 10^3}{\left(\frac{\pi D^4}{32}\right)} = \frac{1}{\frac{D}{2}}$$

$$\therefore \frac{15 \times 10^3 \times 32}{\pi D^3} = \frac{2}{D}$$

$$\therefore D^3 = \frac{15 \times 10^3 \times 32}{2 \times \pi}$$

$$\therefore D = 4243 \text{ cm}$$

Diameter based on angle of twist :

$$\frac{T}{J} = \frac{G\theta}{L}$$

$$\therefore \frac{15 \times 10^3}{\left(\frac{\pi D^4}{32}\right)} = \frac{8 \times 10^3 \times \frac{1}{180}}{210}$$

$$\therefore \frac{15 \times 10^3 \times 32}{\pi D^4} = \frac{8 \times 10^3 \times 1}{180 \times 210}$$

$$D^4 = \frac{15 \times 10^3 \times 32 \times 180 \times 210}{8 \times 10^3 \times \pi}$$

$$D = 2189 \text{ cm}$$

So minimum diameter will be 21.89 cm.

Q.2(e). The motion of a particle is given by $a = t^3 - 3t^2 + 5$, where a is the acceleration in m/sec² and t is the time in seconds. The velocity of the particle at $t = 1$ sec is 6.25 m/sec, and the displacement is 8.30 meters. Calculate the displacement and the velocity at $t = 2$ sec.

Ans. Condition given are

at $t = 1 \text{ sec}$ $v = 6.25 \text{ m/sec}$ $x = 8.30 \text{ m}$

$$a = t^3 - 3t^2 + 5$$

$$a = \frac{dv}{dt}$$

$$= t^3 - 3t^2 + 5$$

$$dv = (t^3 - 3t^2 + 5)dt$$

$$v = \frac{t^4}{4} - \frac{3t^3}{3} + 5t + C_1$$

conditions are $t = 1, v = 6.25$

$$\therefore 6.25 = \frac{1}{4} - 1 + 5 + C_1$$

$$C_1 = 2$$

$$\therefore v = \frac{t^4}{4} - t^3 + 5t + 2$$

$$v = \frac{dx}{dt} = \frac{t^4}{4} - t^3 + 5t + 2$$

$$dx = \left(\frac{t^4}{4} - t^3 + 5t + 2 \right) dt$$

$$x = \frac{t^5}{5} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + C_2$$

Conditions are $t = 1, x = 8.30$

$$\therefore 8.30 = \frac{1}{5} - \frac{1}{4} + \frac{5}{2} + 2 + C_2$$

$$C_2 = 3.85$$

$$\therefore x = \frac{t^5}{5} - \frac{t^4}{4} + \frac{5t^2}{2} + 2t + 3.85$$

\therefore Displacement (x) at $t = 2$ we get

$$x = \frac{2^5}{5} - \frac{2^4}{4} + \frac{5}{2} \times 2^2 + 2 \times 2 + 3.85$$

$$= 20.25 \text{ m}$$

Velocity (v) at $t = 2$ we get

$$v = \frac{2^4}{4} - 2^3 + 5 \times 2 + 2$$

$$= 8 \text{ m/sec.}$$

Section-C

Q. 3. Answer any two parts of the following :

(2 × 5 = 10)

- (a) State and prove Varignon's theorem.
- (b) Derive an expression for the ratio of belt tensions in a flat belt drive.
- (c) Explain briefly different types of friction.

Ans. (a) Varignons theorem : It states that the algebraic sum of the moments of two forces about any point in their plane is equal to the moment of their resultant about point.

Consider force P acting at a point A and having components P_1 and P_2 in any two directions.

Let us consider any point 'O' lying in the plane of the forces, as a moment centre.

Now, Moment of force P about O will be

$$\begin{aligned} Pd &= P(OA \cos \theta) \\ &= OA(P \cos \theta) = OA \times x \end{aligned} \quad \dots(1)$$

Moment of force P_1 about O

$$\begin{aligned} P_1 d_1 &= P_1 (OA \cos \theta_1) \\ &= OA(P_1 \cos \theta_1) = OA \times x_1 \end{aligned} \quad \dots(2)$$

$$\begin{aligned} P_2 d_2 &= P_2 (OA \cos \theta_2) \\ &= OA(P_2 \cos \theta_2) = OA \times x_2 \end{aligned} \quad \dots(3)$$

Adding equation (2) and (3)

$$P_1 d_1 + P_2 d_2 = OA(x_1 + x_2)$$

But

$$x = x_1 + x_2$$

The sum of x component of the forces P_1 and $P_2 = x$ component of the resultant P .

$$\therefore OA \times x = P_1 d_1 + P_2 d_2$$

From equation (1)

$$OA \times x = Pd$$

$$\therefore OA \times x = P_1 d_1 + P_2 d_2$$

$$\therefore Pd = P_1 d_1 + P_2 d_2$$

(b) A driven pulley rotating in clockwise direction is shown in figure.

T_1 = Tension on tight side of belt

T_2 = Tension on slack side of belt

θ = angle of contact of belt with pulley

μ = coefficient of friction between the belt and pulley

Consider a small portion of belt AB in contact with the pulley. AB makes angle $\delta\theta$ at centre of pulley. Let T and $(T + \delta T)$ be the tension at extremities A and B .

The belt portion AB is in equilibrium under the action of following forces.

Tension T in belt at A

Tension $T + \delta T$ in belt at B

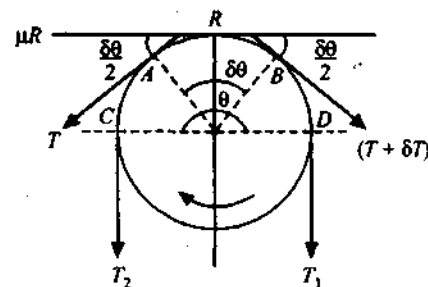
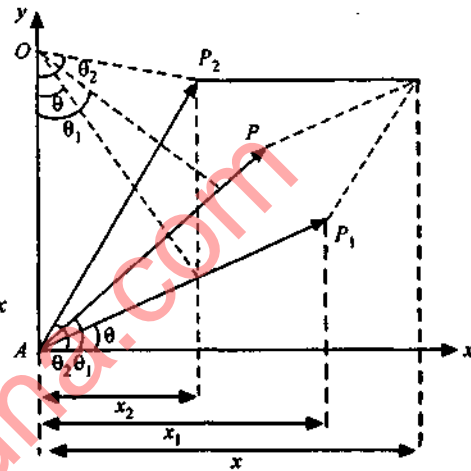
Normal reaction R at C

Frictional forces μR

Now again solving the forces expression in tangential direction

$$\mu R + T \cos\left(\frac{\delta\theta}{2}\right) = (T + \delta T) \cos\frac{\delta\theta}{2}$$

Since $\delta\theta$ is very small $\cos\left(\frac{\delta\theta}{2}\right) = 1$



$$\mu R + T = T + \delta T$$

$$\mu R = \delta T$$

and resolving the force in radial direction

$$R = T \sin\left(\frac{\delta\theta}{2}\right) + (T + \delta T) \sin\left(\frac{\delta\theta}{2}\right)$$

Since $\delta\theta$ is very small, so

$$R = T \frac{\delta\theta}{2} + (T + \delta T) \frac{\delta\theta}{2} = T\delta\theta$$

\therefore

$$\mu(T\delta\theta) = \delta T$$

$$\frac{\delta T}{T} = \mu\delta\theta$$

on integration we get

$$\int_{T_1}^{T_2} \frac{\delta T}{T} = \int_0^\theta \mu d\theta$$

\therefore

$$\ln \frac{T_1}{T_2} = \mu\theta$$

\therefore

$$\frac{T_1}{T_2} = e^{\mu\theta}$$

(c) Friction is divided into two types.

(i) Static friction

(ii) Dynamic friction

The forces of friction developed between the surface of contact of two bodies in the position of rest is known as static friction.

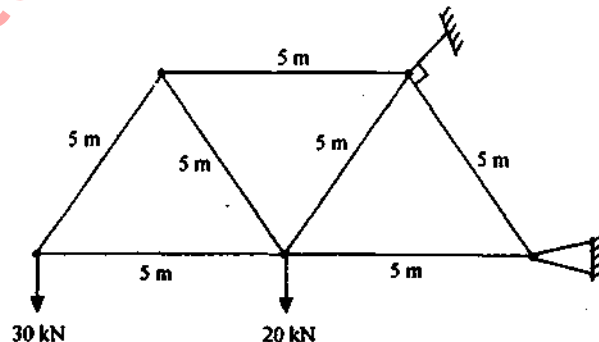
The force of friction developed between the surfaces of contact of two bodies in motion is known as dynamic friction.

If the surfaces are dry (not lubricated) then friction between surfaces is known as solid friction.

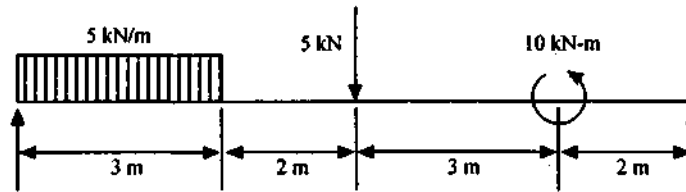
Q. 4. Answer any one part of the following :

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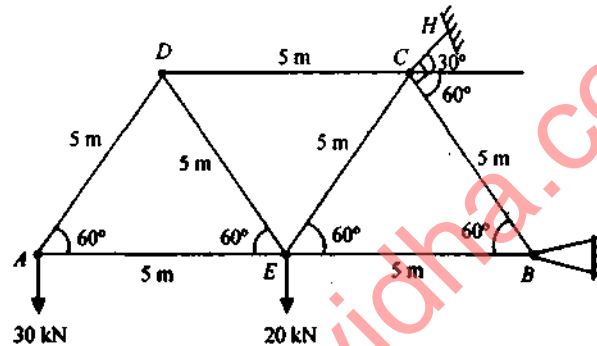
(a) Find the axial forces in all members of a truss as shown in Fig.



(b) Draw the shear force and bending moment diagram for the beam loaded as shown in Fig.

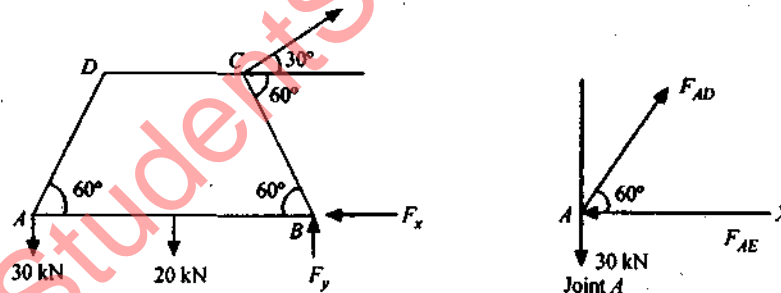


Ans. (a)



The inclination of CH is obtained from geometry.

Free body diagrams (a) and (b) are drawn to represent the forces. Name the members by Bow's method.



$$\begin{aligned}\sum F_x &= 0 & T \cos 30^\circ - F_x &= 0 \\ \sum F_y &= 0 & T \sin 30^\circ + F_y - 20 - 30 &= 0 \\ \sum M_B &= 0 & T \times 5 - 20 \times 5 - 30 \times 10 &= 0\end{aligned}$$

\therefore

$$\begin{aligned}T &= 80 \text{ kN} \\ F_x &= 80 \cos 30^\circ = 69.28 \text{ kN} \\ F_y &= 50 - 80 \sin 30^\circ = 10 \text{ kN}\end{aligned}$$

For joint A :

$$\begin{aligned}\sum F_x &= 0 \quad \therefore F_{AE} - F_{AD} \cos 60^\circ = 0 \\ \sum F_y &= 0 \quad \therefore F_{AD} \sin 60^\circ - 30 = 0\end{aligned}$$

∴

$$F_{AD} = 34.64 \text{ kN}$$

$$F_{AE} = 17.32 \text{ kN}$$

For joint D :

$$\sum F_x = 0 \quad \therefore$$

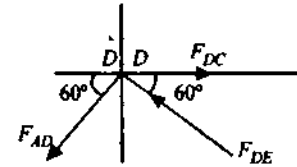
$$\sum F_y = 0 \quad \therefore$$

$$F_{DE} \cos 60^\circ - F_{DC} + F_{AD} \cos 60^\circ = 0$$

$$F_{AD} \sin 60^\circ - F_{DE} \sin 60^\circ = 0$$

$$F_{DE} = \frac{34.64 \sin 60^\circ}{\sin 60^\circ} = 34.64 \text{ kN}$$

$$\begin{aligned} F_{DC} &= F_{AD} \cos 60^\circ + F_{DE} \cos 60^\circ \\ &= 34.64 \cos 60^\circ + 34.64 \cos 60^\circ \\ &= 34.64 \text{ kN} \end{aligned}$$



For Joint E :

$$\sum F_x = 0 \quad \therefore$$

$$\sum F_y = 0 \quad \therefore$$

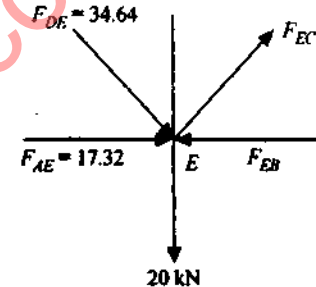
$$-F_{EB} + F_{AE} + F_{EC} \cos 60^\circ + F_{DE} \cos 60^\circ = 0$$

$$F_{DE} \sin 60^\circ + 20 - F_{EC} \sin 60^\circ = 0$$

$$F_{EC} = \frac{34.64 \sin 60^\circ + 20}{\sin 60^\circ}$$

$$= 57.73 \text{ kN}$$

$$\begin{aligned} F_{EB} &= F_{AE} + F_{EC} \cos 60^\circ + F_{DE} \cos 60^\circ \\ &= 17.32 + 57.73 \cos 60^\circ + 34.64 \cos 60^\circ \\ &= 46.18 \text{ kN} \end{aligned}$$



For Joint B :

$$\sum F_x = 0 \quad \therefore$$

$$\sum F_y = 0 \quad \therefore$$

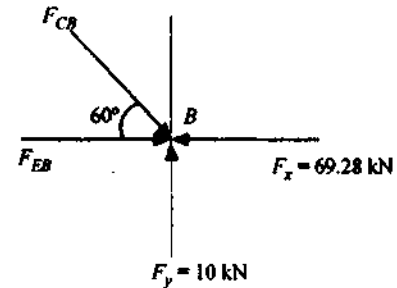
∴

$$F_{EB} - F_x + F_{CB} \cos 60^\circ = 0$$

$$F_y - F_{CB} \sin 60^\circ = 0$$

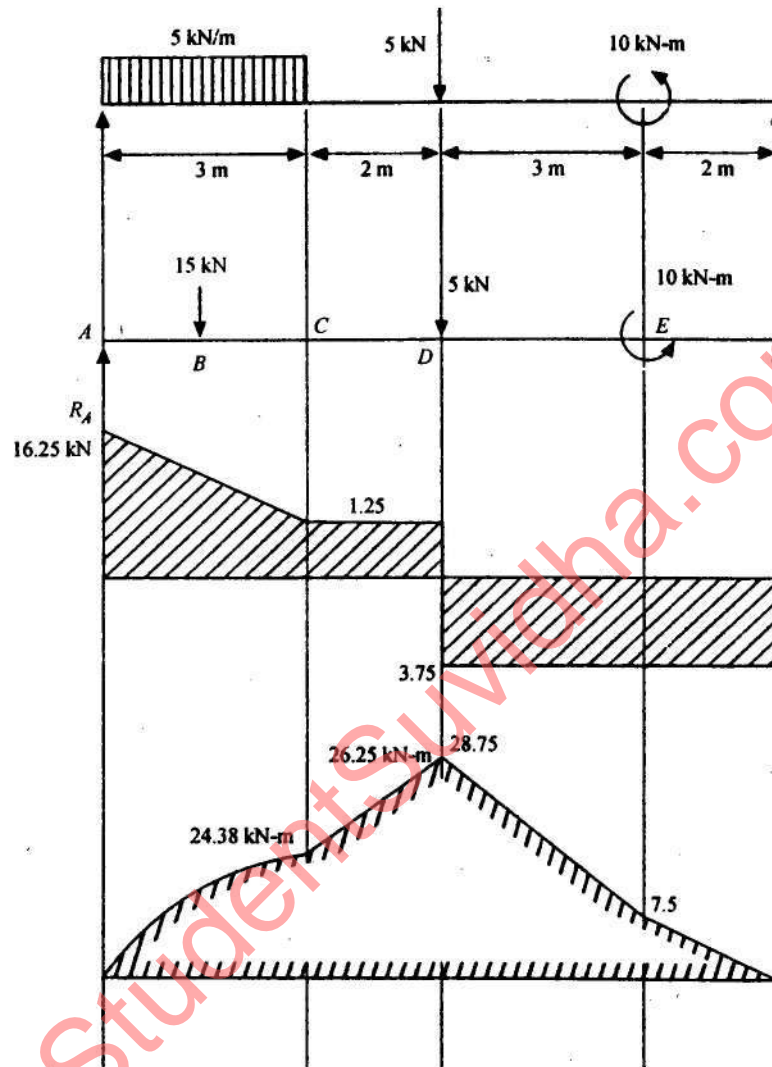
$$F_{CB} = \frac{10}{\sin 60^\circ} = 11.55 \text{ kN}$$

$$\begin{aligned} F_{EB} &= F_x - F_{CB} \cos 60^\circ \\ &= 69.28 - 11.55 \cos 60^\circ \\ &= 63.51 \text{ kN} \end{aligned}$$



| Forces | Magnitude | Nature |
|----------|-----------|--------|
| F_{AD} | 34.64 kN | T |
| F_{AE} | 17.32 kN | C |
| F_{DE} | 34.64 kN | C |
| F_{DC} | 34.64 kN | T |
| F_{EC} | 57.73 kN | T |
| F_{EB} | 46.18 kN | C |
| F_{CB} | 11.55 kN | C |
| F_{EB} | 63.51 kN | C |

(b)



By the help of equilibrium condition

$$\sum F_y = 0 \quad \therefore \quad R_A + R_F - 5 - 15 = 0$$

$$R_A + R_F = 20 \text{ kN}$$

$$\sum M_A = 0 \quad 15 \times 15 + 5 \times 5 - 10 - R_F \times 10 = 0$$

$$R_F = 3.75 \text{ kN}$$

$$R_A = 16.25 \text{ kN}$$

S.F. Calculation :

$$\text{S.F. between } AB = 16.25 \text{ kN}$$

$$\text{S.F. between } BC = 16.25 - 15 = 1.25 \text{ kN}$$

$$\text{S.F. between } CD = 1.25 - 5 = -3.75 \text{ kN}$$

S.F. between $DF = -3.75 + 3.75 = 0$

B.M. Calculation :

B.m. at point $A = 0$

B. m. at point $B = 1625 \times 15 = 24.38 \text{ kN-m}$

B.m. at point $C = 1625 \times 3 - 15 \times 15 = 2625 \text{ kN-m}$

B.m. at point $D = 1625 \times 5 - 15 \times 3.5 = 2875 \text{ kN-m}$

B.m. at point $E = 1625 \times 8 - 15 \times 6.5 - 5 \times 3 - 10 = 7.5 \text{ kN}$

B.m. at point $F = 1625 \times 10 - 15 \times 8.5 - 5 \times 5 - 10 = 0$

Q. 5. Answer any two parts of the following :

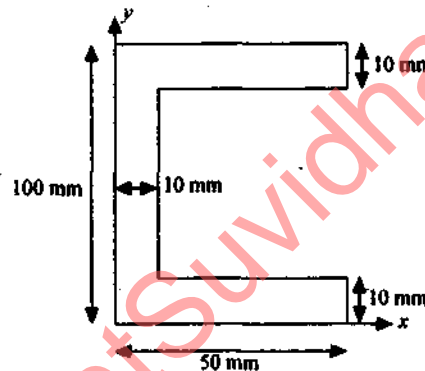
(2 × 5 = 10)

(a) Explain the following :

(i) Product of inertia

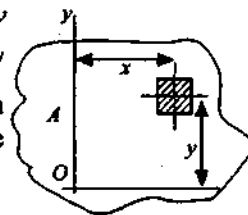
(ii) Mass moment of inertia

(b) Locate the centroid of channel section as shown in Figure.



(c) Determine the mass moment of inertia of a rectangular plate of size $a \times b$ and thickness t about the centroidal axis.

Ans. (a) (i) Product of inertia : Consider a plane figure of area A in the $x-y$ plane. Divide this area into infinitesimal areas. The integral $I_{xy} = \int xy dA$ obtained by multiplying each element dA of the area A by its coordinates x and y and the integration expanding over the entire area of the plane figure is called the product of inertia of the figure with respect to the x and y axes.

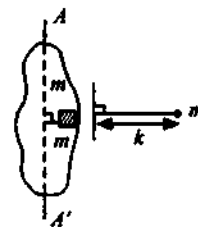


(ii) Mass moment of inertia : Consider a body of mass m . The moment of inertia of the body with respect the axis AA' is defined by integral $I = \int r^2 dm$ where dm

is the mass of an element of the body situated at a distance r from the axis.

The axis AA' and the integration is extended over the entire volume of the body.

The radius of gyration k of the body with respect to the axis AA' is given by the relation



$$I = k^2 m$$

$$k = \sqrt{\frac{I}{m}}$$

(b) Area A_1 of figure $ABCD$ $50 \times 10 = 500 \text{ mm}^2$

Area A_2 of figure $DEFG$ $80 \times 10 = 800 \text{ mm}^2$

Area A_3 of figure $GHIJ$ $50 \times 10 = 500 \text{ mm}^2$

\therefore

$$\bar{X} = \frac{A_1 x_1 + A_2 x_2 + A_3 x_3}{A_1 + A_2 + A_3}$$

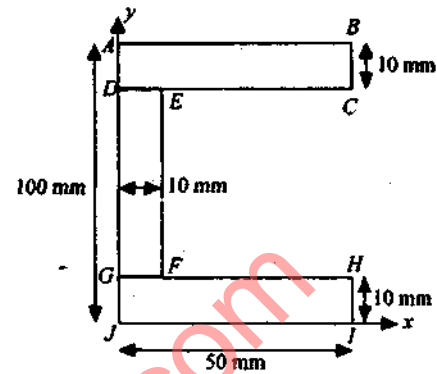
$$= \frac{500 \times 25 + 800 \times 5 + 500 \times 25}{500 + 800 + 500}$$

$$= 16.11 \text{ mm}$$

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3}$$

$$= \frac{500 \times 5 + 800 \times 50 + 500 \times 95}{500 + 800 + 500}$$

$$= 50 \text{ mm}$$



(c) To find I_{xx} :

Consider an elemental strip of width dy at a distance y from x axis

Mass of the element

$$dm = \rho b \times t \times dy$$

(ρ = unit mass of material)

$$I_{xx} = \int_{-a/2}^{a/2} y^2 dm$$

$$= \int_{-a/2}^{a/2} y^2 \rho b t dy$$

$$= \left[\frac{\rho b t y^3}{3} \right]_{-a/2}^{a/2}$$

$$= \frac{\rho b t a^3}{12}$$

But mass of the plate $m = \rho b t a$

$$I_{xx} = \frac{M a^2}{12}$$

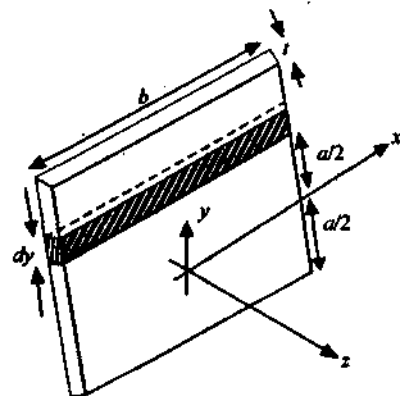
To find I_{yy} : Taking an elemental strip parallel to axis,

$$I_{yy} = \frac{M b^2}{12} \text{ (same as above)}$$

To find I_{zz} , consider an elemental area $dx \times dy$ and thickness

$$r^2 = x^2 + y^2$$

$$I_{zz} = \int r^2 dm$$

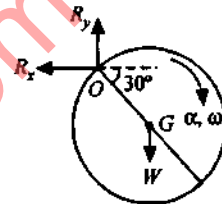


$$\begin{aligned}
 &= \int (x^2 + y^2) dm \\
 &= \int x^2 dm + \int y^2 dm \\
 I_z &= I_{xx} + I_{yy} \\
 &= \frac{ma^2}{12} + \frac{mb^2}{12} \\
 &= \frac{m}{12} (a^2 + b^2)
 \end{aligned}$$

Q. 6. Answer any one part of the following :

(a) A train starts from rest and moves along a curved track of radius 600 m with uniform acceleration until it attains a velocity of 70 km/h at the end of third minute. Determine the tangential, normal and total acceleration of the train at the end of second minute.

(b) The cylinder shown in Figure is 70 cm in diameter and weighs 500 N. It is rotating about the fixed axis O and has an angular velocity of 7 rad/s at the given instant. Using D' Alembert's principle, find the horizontal and vertical components of the reaction at O .



Ans. (a) At $t = 3$ sec, velocities of train $v = 72$ km/hour

$$= \frac{72 \times 10^3}{3600} \text{ m/sec} = 20 \text{ m/sec}$$

$$a_n = \frac{v^2}{r} = \frac{20 \times 20}{600} = 0.67 \text{ m/sec}^2 \quad (\text{Normal acceleration component})$$

$$a_t = \frac{dv}{dt} = \frac{\Delta v}{\Delta t} = \frac{20 - 0}{3} = 6.67 \text{ m/sec}^2$$

$$v_x = \frac{dx}{dt} = \frac{d}{dt} (r \cos \omega t) = -\omega r \sin \omega t$$

$$v_y = \frac{dy}{dt} = \frac{d}{dt} (r \sin \omega t) = \omega r \cos \omega t$$

$$t = 2 \text{ sec}, \omega = \frac{v}{r}$$

$$v = u + at$$

$$\therefore 20 = 0 + a \times 3$$

$$a = \frac{20}{3} = 6.67 \text{ m/sec}^2$$

$$\text{at } t = 2$$

$$v = u + at$$

$$v = 0 + 6.67 \times 2$$

$$= 13.34 \text{ m/sec}$$

a_t = tangential acceleration at $t = 2$

$$a_t = \frac{dv}{dt} = \frac{13.34}{2} = 6.67 \text{ m/sec}^2$$

a_n = normal acceleration at $t = 2$

$$a_n = \frac{v^2}{r} = \frac{(13.34)^2}{600} = 0.29 \text{ m/sec}^2$$

$$\omega = \frac{v}{r} = \frac{13.34}{600} = 0.02 \text{ radium/sec}$$

$$t = 2 \text{ sec}, \omega = 0.02 \text{ rad/sec} = 0.02 \times \frac{180}{\pi} \text{ degrees/sec } r = 600 \text{ m}$$

$$v_x = -0.02 \times 600 \sin \left(0.02 \times 2 \times \frac{180}{\pi} \right)$$

$$= 0.48 \text{ m/sec}$$

$$v_y = 0.02 \times 600 \cos \left(0.02 \times 2 \times \frac{180}{\pi} \right)$$

$$= 1.199 \text{ m/sec}$$

$$a_x = -\omega^2 r \cos \omega t$$

$$= -(0.02)^2 \times 600 \times \cos \left(0.02 \times 2 \times \frac{180}{\pi} \right)$$

$$= -0.24 \text{ m/sec}^2$$

$$a_y = -\omega^2 r \sin \omega t$$

$$= -(0.02)^2 \times 600 \times \sin \left(0.02 \times 2 \times \frac{180}{\pi} \right)$$

$$= -0.0096 \text{ m/sec}^2$$

Total acceleration

$$(a) = \sqrt{a_x^2 + a_y^2}$$

$$= \sqrt{(0.24)^2 + (0.0096)^2}$$

$$= 0.24 \text{ m/sec}^2$$

Ans. (b) Diameter of cylinder $D = 70 \text{ cm}$

moment of inertia of the body

$$I = \frac{1}{2} Mr^2 = \frac{1}{2} \times \frac{500}{9.8} \times \left(\frac{35}{100} \right)^2 = 4.25$$

$\therefore r =$ radius of the cylinder

$$= 35 \text{ cm} = 0.35 \text{ m}$$

Moment about G

$$R_y \times r \cos 30^\circ - R_x \times r \sin \theta - I\alpha = 0$$

By equilibrium condition

$$W = R_y$$

\therefore

$$R_y = 500 \text{ N}$$

R_x and R_y are the horizontal and vertical component.

$$500 \times 0.35 \cos 30^\circ - R_x \times 0.35 \sin 30^\circ - 4.25 \times 7 = 0$$

$$R_x = 121.8 \text{ N}$$

Q. 7. Answer any two of the following :

(a) A 300 mm deep rectangular beam is simply supported over a span of 6 m. What uniformly distributed load per meter the beam can carry if bending stress is not to exceed 110 N/mm². Take $I = 8.5 \times 10^6 \text{ mm}^2$.

(b) A rectangular bar of uniform cross-section 4 cm × 2.5 cm and of length 2.2 m is hanging vertically from a rigid support. It is subjected to axial tensile loading of 10 kN. If density of steel is 8000 kg/m³ and $E = 200 \text{ GN/m}^2$, find the maximum stress and the elongation of the bar.

(c) Derive the torsion formula $\frac{T}{J} = \frac{\tau}{r} = \frac{G\theta}{l}$.

Ans. (a) $\frac{M}{I} = \frac{f_b}{y}$

$$f_b = 110 \text{ N/mm}^2, \quad I = 8.5 \times 10^6 \text{ mm}^2, \quad y = \frac{300}{2} \text{ mm}$$

$$\begin{aligned} \therefore M &= \frac{f_b I}{y} \\ &= \frac{110 \times 8.5 \times 10^6}{150} \\ &= 623 \times 10^6 \text{ N-mm} \end{aligned}$$

For U.D.L. maximum bending moment is $\frac{\omega L^2}{8} = 623 \times 10^6$

$$L = 6 \text{ m} = 6 \times 10^3 \text{ mm}$$

$$\therefore \frac{\omega \times 6 \times 10^3}{8} = 623 \times 10^6$$

$$\begin{aligned} \therefore \omega &= \frac{623 \times 10^6 \times 8}{6 \times 10^3} \\ &= 831 \times 10^3 \text{ N/mm} \end{aligned}$$

$\omega = 831 \text{ kN/mm}$ is the value of uniformless distributed load.

(b) $A = (4 \times 2.5) \text{ cm}^2$
 $= 10 \text{ cm}^2 = 10 \times (10^{-2})^2 \text{ m}^2$
 $= 10 \times 10^{-4} \text{ m}^2$
 $L = 2.2 \text{ m}$

Tensile stress $(\sigma_t) = \frac{F}{A} = \frac{10 \times 10^3}{10 \times 10^{-4}}$
 $= \frac{10 \times 10^3 \times 10^4}{10} = 10^7 \text{ N/m}^2$

$$\begin{aligned} \text{Self weight of the material} &= 8000 \times (4 \times 2.5) \times 2.2 \times 10^{-4} \text{ kg} \\ &= 17.6 \text{ kg} \times g = 172.48 \text{ N} \end{aligned}$$

\therefore Maximum tensile stress $= \sigma_{\max}$

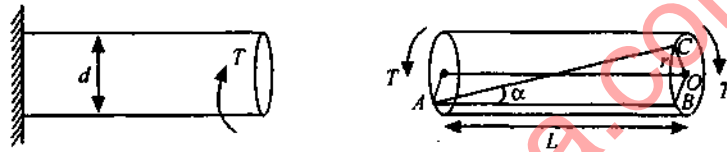
$$= \frac{17248 + 10 \times 10^3}{(10 \times 10^{-4})} \text{ N/m}^2$$

$$E \text{ elongation } (\Delta) = \frac{F \times L^2}{2E}$$

$$= \frac{(17248 + 10 \times 10^3) \times (22)^2}{2 \times 200 \times 10^9} \text{ m}$$

$$= 123 \times 10^{-7} \text{ m}$$

(c) A circular shaft is subjected to pure torque 'T' about its polar axis, shear stresses set up in directions perpendicular to the radius on all transverse sections. The complementary shear stress on longitudinal plane will cause a distortion of filaments which are originally in the longitudinal direction.



A generator on the surface of the shaft denoted by AB, deforms into the configuration AC after torsion has occurred. The angle between these configuration is denoted by α . By definition, the shearing strain on the surface of the shaft is

$$\gamma = \tan \alpha \approx \alpha$$

$$\alpha = \frac{BC}{L} = \frac{r \cdot \theta}{L}$$

\therefore

$$\gamma = \frac{r \cdot \theta}{L}$$

But since a diameter of the shaft before loading is assumed to remain a diameter after torsion, the shearing strain at a general distance ρ from the centre of the shaft is given by $\gamma_\rho = \frac{\rho \cdot \theta}{L}$.

Sum of the moments of the distributed shearing forces over the entire circular cross section = Applied twisting moment

$$\Rightarrow \int_0^r \tau_\rho \rho \cdot da = T$$

$$= \frac{\tau_\rho}{\rho} = \frac{\tau}{r}$$

$$= \text{constant}$$

Consequently, dividing and multiplying in ρ in right hand side of equation

$$T = \int_0^r \frac{\tau_\rho}{\rho} (\rho^2) da$$

$$T = \frac{\tau_\rho}{\rho} \int_0^r \rho^2 \cdot da$$

$$T = \frac{\tau_\rho}{\rho} \int_0^r \rho^2 \cdot 2\pi \rho d\rho$$

$$\begin{aligned}
 &= \frac{\tau_p}{\rho} \int_0^r 2\rho^3 \cdot d\rho \\
 &= \frac{\tau}{r} 2z \left| \frac{\rho^4}{4} \right|_0^r \\
 &= \frac{\tau}{r} \frac{\pi}{2} r^4 \\
 &= \frac{2}{16} \tau \cdot d^3
 \end{aligned}$$

Since the ratio $\frac{\tau_p}{\rho}$ is a constant and the quantities $\int_0^r \rho^2 \cdot da$ is called polar moment of inertia by definition, denoted by J hence,

$$T = \frac{\tau_p \cdot J}{\rho}$$

Now for outermost fiber where $\rho = r$

$$\gamma_r = \frac{r \cdot \theta}{2}$$

and

$$\tau = \frac{T \cdot r}{J} \text{ or } \frac{T}{J} = \frac{\tau}{r}$$

By definition of modulus of elasticity $G = \frac{\tau}{\gamma}$

$$G = \frac{\tau}{\gamma} = \frac{T \cdot r}{J} \cdot \frac{r \cdot \theta}{L} = \frac{T \cdot L}{J \cdot \theta}$$

$$G = \frac{T \cdot L}{J \cdot \theta}$$

$$J = \frac{T \cdot L}{G \cdot \theta}$$

or

$$\frac{T}{J} = \frac{G \cdot \theta}{L} = \frac{\tau}{r}$$